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Dynamic problem of generalized thermoelastic diffusive medium†

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Abstract

The equations of generalized thermoelastic diffusion, based on the theory of Lord and Shulman with one relaxation time, are derived for anisotropic media with rotation. The variational principle and reciprocity theorem for the governing equations are derived. The propagation of leaky Rayleigh waves in a viscous fluid layer overlying a homogeneous isotropic, generalized thermoelastic diffusive half-space with rotating frame of reference is studied.

Keywords: Wave propagation; Rotation; Viscous fluid; Phase velocity; Attenuation coefficient

1. Introduction

The study of the interaction of elastic waves with fluid-loaded solids has been recognized as a viable means for non-destructive evaluation of solid structures. Qi [1] studied the influence of viscous fluid loading on the propagation of leaky Rayleigh wave in the presence of heat conduction effects. The spontaneous movement of the particles from high concentration region to the low concentration region is defined as diffusion and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position. The thermo diffusion in elastic solids is due to coupling of fields of temperature, mass diffusion and that of strain in addition to heat and mass exchange with environment.

This article deals with the generalized thermoelastic diffusion problem with one relaxation time for anisotropic media in rotating frame of reference. The propagation of leaky Rayleigh waves in a viscous fluid layer overlying a homogeneous isotropic, thermoelastic diffusive half-space with rotating frame of reference in the context of generalized theoryies of thermoelastic diffusion is investigated. The phase velocity and attenuation coefficient of these waves are presented graphically to depict the effect of viscosity.

2. Basic Equations

The basic governing equations for an anisotropic, homogeneous elastic solid with generalized thermoelastic diffusion in rotating frame of reference are:

$$\sigma_{ij,j} + \rho F_i = \rho[\ddot{u}_i + \{\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u})\}_i + (2\mathbf{\Omega}\times\dot{\mathbf{u}})_i],$$
$$\sigma_{ij} = c_{ijkl}e_{kl} + a_{ij}T + b_{ij}C,\ -q_{i,i} = \rho T_0\dot{S},$$
$$-K_{ij}T_{,j} = q_i + \tau_0\dot{q}_i,\ \rho ST_0 = \rho C_E T - a_{ij}e_{ij}T_0 + aCT_0,$$
$$\frac{\partial \alpha^*_{ij}P_{,j}}{\partial x_i} = \dot{C} + \tau^0\ddot{C}\ , P = b_{ij}e_{ij} + bC - aT\ , \quad (1)$$

Here, the medium is rotating with angular velocity, $\mathbf{\Omega} = \Omega\hat{y}$ where $\hat{y}$ is the unit vector along the axis of rotation and the equations of motion include two additional terms: the centripetal acceleration $\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u})$ due to time-varying motion, and the Coriolis acceleration $2\mathbf{\Omega}\times\dot{\mathbf{u}}$. The remaining symbols have their usual similar meanings as in Aouadi [2]. For a coupled thermoelastic diffusion problem, $\tau_0 = \tau^0 = 0$.

Equations (ii), (v) and (vii) of (1) can be rewritten as:

$$\sigma_{ij} = d_{ijkl}e_{kl} + s_{ij}T + \varsigma_{ij}P,\ \rho S = \kappa T - s_{ij}e_{ij} + nP\ ,$$
$$C = \varepsilon P - \varsigma_{kl}e_{kl} + nT\ , \quad (2)$$

where

$$s_{ij} = a_{ij} + \frac{ab_{ij}}{b}, d_{ijkl} = c_{ijkl} - \frac{b_{ij}b_{kl}}{b}, \varsigma_{ij} = \frac{b_{ij}}{b},$$
$$\kappa = \frac{\rho C_E}{T_0} + \frac{a^2}{b}, \varepsilon = \frac{1}{b}, n = \frac{a}{b}.$$

3. Variational principle

The principle of virtual work with variation of displacements (the virtual work of body forces F_i, inertial forces $\rho\dot{u}_i$, centripetal forces $\rho\{\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u})\}_i$, Coriolis forces $\rho(2\mathbf{\Omega}\times\dot{\mathbf{u}})_i$,

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surface forces $h_i = \sigma_{ji} n_j$ and the virtual work of internal forces) for the elastic deformable body is written as

$$\int_V \rho[X_i - \ddot{u}_i]\delta u_i dV + \int_A h_i \delta u_i dA = \int_V \sigma_{ji} \delta u_{i,j} dV,$$
$$X_i = F_i - \{\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u})\}_i - (2\mathbf{\Omega} \times \dot{\mathbf{u}})_i \quad (3)$$

We define a vector J connected with the entropy through the relation $\rho S = -J_{i,i}$. Using this relation and from eqns. (iii-iv) of (1), we obtain

$$\frac{T_0}{K_{ij}}\left(\frac{d}{dt} + \tau_0 \frac{d^2}{dt^2}\right) J_i + T_{,j} = 0 \quad (4)$$

Multiplying (4) by δJ_i and integrating over the region of the body and using the divergence theorem with the aid of (ii) of eqn. (2) and above relation, we get

$$\int_A T n_j \delta J_j dA - \int_V s_{ij} T \delta e_{ij} dV + n \int_V T \delta P dV + \delta(G + M) = 0$$
$$G = \frac{\kappa}{2}\int_V T^2 dV, M = \frac{1}{2}\int_V \frac{T_0}{K_{ij}}\left(\frac{d}{dt} + \tau_0 \frac{d^2}{dt^2}\right) J_i \delta J_j dV$$

Now we introduce the vector function N defined as $C = -N_{i,i}$. Using this relation and from eqns. (vi) of (1), we obtain

$$\frac{1}{\alpha_{ij}^*}\left(\frac{d}{dt} + \tau^0 \frac{d^2}{dt^2}\right) N_i + P_{,j} = 0 \quad (5)$$

Multiplying (5) by δN_i and integrating over the region of the body and using the divergence theorem with the aid of (iii) of eqn. (2) and above relation, we get

$$\int_A P n_j \delta N_j dA - \int_V \varsigma_{ij} P \delta e_{ij} dV + n \int_V P \delta T dV + \delta(X + Y) = 0$$
$$X = \frac{\varepsilon}{2}\int_V P^2 dV, Y = \frac{1}{2}\int_V \frac{1}{\alpha_{ij}^*}\left(\frac{dN_i}{dt} + \tau^0 \frac{d^2 N_i}{dt^2}\right) \delta N_j dV$$

Using above two derived equations, eqns. (2) and (3), we obtain the variational principle in the final form

$$\delta(W + G + M + X + Y + n\int_V PTdV) =$$
$$\int_V \rho[X_i - \ddot{u}_i]\delta u_i dV + \int_A h_i \delta u_i dA - \int_A P n_i \delta N_i dA - \int_A T n_i \delta J_i dA$$

where $W = \frac{1}{2}\int_V d_{ijkl} e_{ij} e_{kl} dV$.

Now we assume that the virtual displacements δu_i, the virtual increment of the temperature δT, etc. correspond to the increments occurring really in the body in question. Then

$$\delta u_i = \frac{\partial u_i}{\partial t} dT = \dot{u}_i dT, \delta T = \frac{\partial T}{\partial t} dT = \dot{T} dT, \text{ etc.}$$

and the above equation is reduced to the following relation

$$\frac{d}{dt}(W + G + M + X + Y + n\int_V PTdV) =$$
$$\int_V \rho[X_i - \ddot{u}_i]\dot{u}_i dV + \int_A h_i \dot{u}_i dA - \int_A P n_i \dot{N}_i dA - \int_A T n_i \dot{J}_i dA$$

The uniqueness theorem can be proved on the basis of Aouadi [2].

4. Reciprocity theorem

We shall consider a homogeneous anisotropic generalized thermoelastic diffusive body occupying the region V and bounded by the surface A. We assume that the stresses σ_{ij} and the strains e_{ij} are continuous together with their first derivatives whereas u_i, T, C, P are continuous and have continuous derivatives up to the second order, for $\mathbf{x} \; \varepsilon \; V + A, t > 0$.

The components of surface traction, the normal component of the heat flux and the normal component of the chemical flux at regular points of ∂V, are given by

$$h_i = \sigma_{ji} n_j, q = K_{ij} T_{,j} n_i, p = \alpha_{ij}^* P_{,j} n_j \quad (6)$$

respectively. We denote by n_j the outward unit normal of ∂V. To the system of field equation, we must adjoin boundary conditions and initial conditions. We consider boundary conditions as:

$u_i(\mathbf{x},t) = U_i(\mathbf{x},t), T(\mathbf{x},t) = \phi(\mathbf{x},t), P(\mathbf{x},t) = \psi(\mathbf{x},t),$ for all $\mathbf{x} \; \varepsilon \; A, t > 0$ and homogeneous initial conditions:

$u_i(\mathbf{x},0) = \dot{u}_i(\mathbf{x},0) = T(\mathbf{x},0) = \dot{T}(\mathbf{x},0) = P(\mathbf{x},0) = \dot{P}(\mathbf{x},0) = 0,$ for all $\mathbf{x} \; \varepsilon \; V, t = 0$

We derive the dynamic reciprocity relationship for a generalized thermodiffusion bounded body V, which satisfies eqs. (1) and (2), the boundary conditions and the homogeneous initial conditions, and subjected to the action of body forces $F_i(\mathbf{x},t)$, surface traction $h_i(\mathbf{x},t)$, centrifugal forces $\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u})$ and Coriolis forces $2\mathbf{\Omega} \times \dot{\mathbf{u}}$, the heat flux $q(\mathbf{x},t)$ and the chemical flux $p(\mathbf{x},t)$.

The Laplace integral transform is defined as

$$\bar{f}(\mathbf{x},s) = L(f(\mathbf{x},t)) = \int_0^\infty f(\mathbf{x},t) e^{-st} dt \quad (7)$$

We now consider two problems where applied body forces, chemical potential and the surface temperature are specified differently. Let the variables involved in these two problems be distinguished by superscripts in parentheses. Thus, we have $u_i^{(1)}$, $e_{ij}^{(1)}$, $\sigma_{ij}^{(1)}$, $T^{(1)}, P^{(1)}$..... for the first problem and $u_i^{(2)}$, $e_{ij}^{(2)}$, $\sigma_{ij}^{(2)}$, $T^{(2)}, P^{(2)}$..... for the second problem. Each set of variables satisfies the system of Eqs. (1), (2), the boundary conditions and the homogeneous initial conditions. Using the strain-displacement relation, the assumption $\sigma_{ij} = \sigma_{ji}$ and the divergence theorem with the aid of (i) of (1), (6) and (7), we obtain

$$\int_V \sigma_{ij}^{(1)} e_{ij}^{(2)} dV = \int_A h_i^{(1)} u_i^{(2)} dA - \rho \int_V s^2 u_i^{(1)} u_i^{(2)} dV +$$
$$\rho \int_V [F_i^{(1)} - \{\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u})\}_i^{(1)} - (2\mathbf{\Omega} \times s\dot{\mathbf{u}})_i^{(1)}] u_i^{(2)} dV$$

A similar expression is obtained for the integral $\int_V \sigma_{ij}^{(2)} e_{ij}^{(1)} dV$, from which together with the above equation, it follows that

$$\int_V [\sigma_{ij}^{(1)} e_{ij}^{(2)} - \sigma_{ij}^{(2)} e_{ij}^{(1)}] dV = \int_A [h_i^{(1)} u_i^{(2)} - h_i^{(2)} u_i^{(1)}] dA +$$
$$\rho \int_V [Y_i^{(1)} u_i^{(2)} - Y_i^{(2)} u_i^{(1)}] dV,$$
$$Y_i^{(g)} = F_i^{(g)} - \{\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u})\}_i^{(g)} - (2\mathbf{\Omega} \times s\dot{\mathbf{u}})_i^{(g)}, g = 1, 2$$

Now multiplying $e_{ij}^{(2)}$ by the corresponding equation (i) of (2) for the first problem, $e_{ij}^{(1)}$ by the analogous equation for the second problem, subtracting and integrating over the region V and using the symmetry properties of d_{ijkl}, we obtain

$$\int_V [\sigma_{ij}^{(1)} e_{ij}^{(2)} - \sigma_{ij}^{(2)} e_{ij}^{(1)}] dV = \int_V s_{ij} [T^{(1)} e_{ij}^{(2)} - T^{(2)} e_{ij}^{(1)}] dV +$$
$$\int_V \varsigma_{ij} [P^{(1)} e_{ij}^{(2)} - P^{(2)} e_{ij}^{(1)}] dV$$

From above two equations, we get the first part of the reciprocity theorem. Following Aouadi [2],we can obtain the second part of reciprocity theorem.

Eliminating integrals $\int_V s_{ij} [T^{(1)} e_{ij}^{(2)} - T^{(2)} e_{ij}^{(1)}] dV$,

$$\int_V \varsigma_{ij} [P^{(1)} e_{ij}^{(2)} - P^{(2)} e_{ij}^{(1)}] dV, \int_V n [P^{(2)} T^{(1)} - P^{(1)} T^{(2)}] dV$$

from first and second parts of the reciprocity theorems, we obtain the generalized parts of the reciprocity theorem in the Laplace transform domain.

To invert the Laplace transform in first, second and generalized parts of the reciprocity theorem, we shall use the convolution theorem

$$L^{-1}\{F(s)G(s)\} = \int_0^t f(t-\xi) g(\xi) d\xi = \int_0^t g(t-\xi) f(\xi) d\xi$$

Applying inverse Laplace transform in first, second and generalized parts of the reciprocity theorem, we obtain these parts in final form.

5. Isotropic case

By setting

$$c_{ijkm} = \lambda \delta_{ij} \delta_{km} + \mu(\delta_{ik} \delta_{jm} + \delta_{jk} \delta_{im}), K_{ij} = K \delta_{ij},$$
$$\alpha_{ij}^* = D \delta_{ij}, a_{ij} = -\beta_1 \delta_{ij}, b_{ij} = -\beta_2 \delta_{ij},$$

where λ and μ are Lame's constants, K is the coefficient of thermal conductivity, D is the diffusion coefficient, $(\beta_1, \beta_2) = (3\lambda + 2\mu)(\alpha_t, \alpha_c)$, α_t and α_c are the coefficients of linear thermal and diffusion expansion, in the Eqs. (1), we obtain the basic equations for isotropic, homogeneous elastic solid with generalized thermoelastic diffusion and from which the equations of motion in the rotating frame of reference, equation of heat conduction and equation of mass diffusion can be obtained as given in Kumar et al. [3] based upon Lord-Shulman (L-S) and Coupled (CT) theories of thermoelasticity.

Following Felher [4], the basic equations in a viscous medium are

$$(K^o + \frac{4}{3}\eta \frac{\partial}{\partial t}) \nabla(\nabla.\mathbf{u}^o) - \eta \frac{\partial}{\partial t} \nabla \times (\nabla \times \mathbf{u}^o) = \rho^o \ddot{\mathbf{u}}^o$$
$$\pi_{ij} = (K^o - \frac{2}{3}\eta \frac{\partial}{\partial t}) \mathbf{u}_{k,k}^o \delta_{ij} + \eta \frac{\partial}{\partial t} (u_{i,j}^o + u_{j,i}^o)$$

Where ρ^o is the fluid density at rest. π_{ij} are the components of force stress tensor in the viscous medium, $\mathbf{u}^o = (u_1^o, u_2^o, u_3^o)$ is the displacement vector in the viscous medium, $c^o = \sqrt{K^o / \rho^o}$ is the sound velocity, K^o and η are respectively bulk modulus and fluid viscosity.

6. Formulation and solution of the problem

We consider a viscous compressible fluid layer of thickness H overlying a homogeneous isotropic, generalized thermoelastic diffusive half-space in rotating frame of reference. The origin of the Cartesian coordinate system (x_1, x_2, x_3) is taken at any point on the plane surface (interface) and x_3-axis points vertically downwards into the solid half-space which is thus represented by $x_3 \geq 0$. In this paper, the problem of symmetric with respect to x_3-axis is considered, so that, all variables are independent of x_2.

We define the following dimensional quantities:

$$\mu^* = \frac{\mu}{\beta_1 T_0}, \eta^* = \frac{\eta \omega_1^*}{\rho_f c_1^2}, \lambda^* = \frac{\lambda}{\beta_1 T_0}, \pi_{ij}^{'} = \frac{\pi_{ij}}{\beta_1 T_0},$$

$$\rho^{o*} = \frac{\rho^o c_1^2}{\beta_1 T_0}, (u_1^{o'}, u_3^{o'}) = \frac{(u_1^o, u_3^o) w_1^*}{c_1}, \eta_1^* = \eta^* \rho^{o*},$$

Where w_1^* and c_1 are the characteristic frequency and the longitudinal wave velocity respectively.

For viscous fluid, we have

$$u_1^o = \frac{\partial \phi^o}{\partial x_1} - \frac{\partial \psi^o}{\partial x_3}, u_3^o = \frac{\partial \phi^o}{\partial x_3} + \frac{\partial \psi^o}{\partial x_1} \tag{8}$$

Where, ϕ^o and ψ^o are scalar velocity potential and vector velocity potential components along the x_2-axis.

The remaining dimensionless quantities can be taken from Kumar et al. [3]. Using dimensionless quantities and (8) in the basic equations of both isotropic thermoelastic diffusive half-space and viscous fluid and assuming the solution $(u_1, u_3, T, C, \phi^o, \psi^o) = (1, L, S, R, E, F) U e^{\iota\xi(x_1 + mx_3 - ct)}$ where $c = \frac{\omega}{\xi}$, is the dimensionless phase velocity, ω is the frequency and ξ is the complex wave number; m is still unknown parameter. $1, L, S, R$ are, respectively, the amplitude ratios of displacements u_1, u_3, T and C with respect to u_1, the expressions for u_1, u_3, T, C, ϕ^o and ψ^o are obtained as

$$(u_1, u_3, T, C) = \sum_{i=1}^{4} (1, n_i, k_i, f_i) A_i \exp(\iota\xi(x_1 + mx_3 - ct)),$$

$$\phi^o = [A_5 \cos(\xi m_5 x_3) + A_6 \sin(\xi m_5 x_3)] \exp(\iota\xi(x_1 - ct)), \tag{9}$$

$$\psi^o = [A_7 \cos(\xi m_6 x_3) + A_8 \sin(\xi m_6 x_3)] \exp(\iota\xi(x_1 - ct)),$$

where $m_i^2, i = 1,2,3,4$ are the roots of the polynomial equation:

$$m^8 + A^* m^6 + B^* m^4 + C^* m^2 + D^* = 0,$$

Where the coefficients A^*, B^*, C^* and D^* are given in the appendix. The roots m_5, m_6 are also given in the appendix.

7. Boundary conditions

The boundary conditions at $x_3 = -H$ and at solid-fluid interface $x_3 = 0$ are as follows:

$$\pi_{33} = \pi_{31} = 0 \quad \text{at} \quad x_3 = -H$$

$$\sigma_{33} = \pi_{33}, \quad \sigma_{31} = \pi_{31}, \quad \dot{u}_1 = \dot{u}_1^o, \ \dot{u}_3 = \dot{u}_3^o \ ,$$

$$T_{,3} = C_{,3} = 0 \quad \text{at} \quad x_3 = 0 .$$

8. Derivation of secular equations

Invoking the boundary conditions and using Eqs. (9), we obtain the secular equation for leaky Rayleigh waves propagating in a viscous fluid layer overlying isotropic thermoelastic diffusive half-space with rotation as

$$R_1 M_1 - R_2 M_2 - R_3 M_3 - R_4 M_4 = 0 \ ,$$

Where R_i, $M_i, i = 1,2,3,4$ are given in appendix.

Case-I: In absence of rotation effect, we obtain the dispersion equation for leaky Rayleigh waves propagating in a viscous fluid layer overlying isotropic thermoelastic diffusive half-space.

Case-II: For $H \to \infty$, we obtain the secular equation for leaky Rayleigh waves at solid-fluid interface $x_3 = 0$ with rotation.

Case-III: For $H \to 0$, we obtain the secular equation for Rayleigh waves propagating in isotropic thermoelastic diffusive half-space with rotation.

9. Numerical results and discussion

We now represent for numerical results for copper material (thermoelastic diffusive solid), the physical data is given below:

λ = 7.76×10^{10} kg·m^{-1}·s^{2}, μ = 3.86×10^{10} kg·m^{-1}·s^{2},
T_0 = 0.293×10^{3} K, C_E = 0.3831×10^{3} J·kg^{-1}·K^{-1},
α_t = 1.78×10^{-5} K^{-1}, α_c = 1.98×10^{-4} m^{3}·kg^{-1},
a = 1.2×10^{4} m^{2}·s^{-2}·K^{-1}, b = 9×10^{5}kg·m^{5}·s^{-2},
τ_0 = 0.25s, τ^0 = 0.15s, D=0.85×10^{-8} g·m^{-3}·s,
ρ = 8.954×10^{3} kg·m^{-3}, K = 0.386×10^{3} W·m^{-1}·K^{-1} , Ω =12 rpm.

For viscous fluid (kerosene and seawater), we take the numerical data from White [5].

In Figs. 1 and 2, the solid and dash lines correspond to the CT theory of thermoelastic diffusion for first and second modes for the case of kerosene (CT1K and CT2K), respectively. The diamond and square symbols, respectively, on these lines correspond to the CT theory of thermoelastic diffusion for first and second modes for the case of seawater (CT1S and CT2S). Similarly, the star and lower triangles, respectively, on these lines correspond to the L-S theory of thermoelastic diffusion for first and second modes for the case of kerosene (LS1K and LS2K). The triangle and circle symbols, respectively on solid and dash lines correspond to the L-S theory of thermoelastic diffusion for first and second modes for the case of seawater (LS1S and LS2S).

From Fig. 1, we observe that in both the cases of kerosene and seawater, the values of phase velocity corresponding to CT1K, CT2K, LS1K and LS2K increase. The values of phase velocity for the second mode are higher than that of first mode in both the cases of CT and L-S theories. The values of phase velocity for CT theory are more as compared to L-S theory for both modes. If we compare both the cases, we find that the increase in the phase velocity is more in the case of seawater than that of kerosene. Fig. 2 shows that the values of attenua-

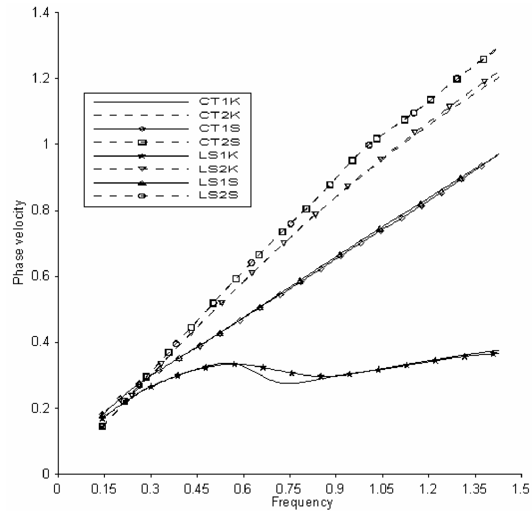

Fig. 1. Variations of phase velocity w.r.t frequency.

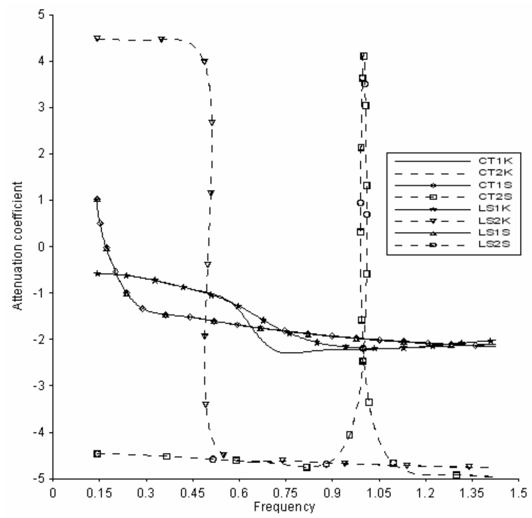

Fig. 2. Variations of attenuation coefficient w.r.t. frequency.

tion coefficient vibrate in all the cases. The values of attenuation coefficient for the case of seawater increase more as compared to the case of kerosene. For CT theory, the values increase slightly higher than that of L-S theory in both the cases of kerosene and seawater.

10. Conclusions

The generalized model for thermoelastic diffusion equations for anisotropic media in the rotating frame of reference based on the theory of Lord and Shulman with one relaxation time is given. The propagation of leaky Rayleigh waves in a viscous fluid layer lying over a homogeneous isotropic thermoelastic diffusive half-space with rotating frame of reference has been considered. The phase velocity and attenuation coefficient of leaky Rayleigh waves are presented graphically.

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Appendix

$$(\tau_t^{10},\tau_f^{10}) = 1-\iota\omega(\tau_0,\tau^0), \Gamma = \frac{\Omega}{\omega}, f_2 = 2\iota\Gamma c^2,$$
$$(f_1,f_3) = (1,\delta_1) - c^2(1+\Gamma^2), f_4 = \zeta_2\tau_t^{10}c, f_5 = \iota\xi$$
$$+c\tau_t^{10}, f_6 = c\zeta_1\tau_t^{10}, f_7 = q_3^* - \iota\omega^{-1}c^2\tau_f^{10}, I_1 = q_1^* -$$
$$q_3^*, I_2 = q_1^* - f_3q_3^*, I_3 = f_5q_3^* + \iota\xi f_7 + f_6q_2^*, I_4 =$$
$$f_6 - f_5, I_5 = q_3^* + q_2^*, I_6 = f_5f_7 + f_6q_2^*, I_7 = f_7 +$$
$$q_2^*, I_8 = q_1^* - \delta_2q_3^*, I_9 = \iota\xi - I_4, I_{10} = f_4q_3^* + q_1^*f_6,$$
$$I_{11} = f_6 + f_4, I_{12} = f_4f_7 + q_1^*f_6, I_{13} = f_7 + q_3^*f_3 -$$
$$q_1^*, I_{14} = I_{11} + f_3f_6, I_{15} = q_1^*(\iota\xi + f_5) - f_4q_2^*, I_{16} =$$
$$f_5 + \iota\xi f_3 + f_4, I_{17} = f_5q_1^* - f_4q_2^*, I_{18} = (f_3+1)q_2^*$$
$$+q_1^*, d_1^* = \iota\xi I_2 - I_3 - q_1^*I_4 - f_4I_5, d_2^* = f_3I_3 + I_6$$
$$+q_1^*I_4 + f_4I_7, d_3^* = q_1^*I_9 - \delta_2I_3 - f_4I_5, d_4^* = \delta_2I_6$$
$$+f_4I_7 + q_1^*I_4, d_5^* = \delta_2I_{10} + f_4I_1 - q_1^*I_{11}, d_6^* =$$
$$\delta_2I_{12} - f_4I_{13} - q_1^*I_{14}, d_7^* = \iota\xi q_1^*(\delta_2 - 1), d_8^* =$$
$$\delta_2I_{15} + f_4(q_1^* + q_2^*) - q_1^*(\iota\xi + I_{16}), d_9^* = \delta_2I_{17} +$$
$$f_4I_{18} - q_1^*(f_3f_5 + I_{16}), d_{10}^* = \iota\xi(f_1I_1 - \delta_2I_8) + \delta_1d_1^* - d_7^*,$$
$$d_{11}^* = f_1d_1^* - \delta_1d_2^* - \delta_2d_3^* - \iota\xi f_2^2q_3^* + d_5^* - d_8^*, d_{12}^* =$$
$$\delta_2d_4^* - f_2^2I_3 - \delta_1f_3I_6 - f_1d_2^* + d_6^* - d_9^*, d_{13}^* =$$
$$f_3(I_{12} + f_1I_6 - I_{17}) + f_2^2I_6, d_{14}^* = \iota\xi\delta_1I_1, \Delta_{1i} =$$
$$\xi I_8m_i^5 + \iota\xi q_3^*f_2m_i^4 + \iota d_3^*m_i^3 - f_2I_3m_i^2 + \iota d_4^*m_i +$$
$$f_2I_6, \Delta_{2i} = d_5^*m_i^4 - \iota f_2I_{10}m_i^3 - d_6^*m_i^2 + \iota f_2I_{12}m_i - f_3I_{12},$$
$$\Delta_{3i} = d_7^*m_i^6 + \xi q_1^*f_2m_i^5 - d_8^*m_i^4 + \iota f_2I_{15}m_i^3 + d_9^*m_i^2 - \iota f_2I_{17}m_i$$
$$+f_3I_{17}, \Delta_{4i} = -\iota\xi I_1m_i^6 + d_1^*m_i^4 + d_2^*m_i^2 - f_3I_6,$$
$$A^* = d_{10}^*/d_{14}^*, B^* = d_{11}^*/d_{14}^*, C^* = d_{12}^*/d_{14}^*, D^* =$$
$$-d_{13}^*/d_{14}^*, n_i = \Delta_{1i}/\Delta_{4i}, k_i = \Delta_{2i}/\Delta_{4i},$$
$$f_i = \Delta_{3i}/\Delta_{4i}, i = 1,2,3,4.$$
$$m_5 = \sqrt{-1 + \frac{3c_1^2c^2}{3c_f^2 - 4\iota\omega\eta^*c_1^2}}, m_6 = \sqrt{-1 + \frac{\iota c}{\xi\eta^*}}$$

$$d_i = \iota\lambda^* - (\lambda^* + 2\mu^*)(m_i n_i + \xi^{-1}(k_i + f_i)),\ h_i = \mu^*(\iota n_i - m_i), i = 1,2,3,4$$

$$c_i = \cos(\xi m_i H), s_i = \sin(\xi m_i H), i = 5,6, R_1 = \frac{(m_6^2 - 1)s_5}{c_5},$$

$$R_2 = \frac{2\iota m_6 s_6}{c_5}, R_3 = \frac{R_1}{\tan(\xi m_5 H)}, R_4 = \frac{2\iota m_6 c_6}{c_5},$$

$$\Delta_1 = [R_3 c_6 \quad 0 \quad -\eta_1^* R_3 \xi \quad 0 \quad -1 \quad 0 \quad 0]^{Tr},$$

$$\Delta_2 = [-R_3 s_6 \quad -2\iota\xi\eta_1^* m_6 \quad 0 \quad -\iota m_6 \quad 0 \quad 0 \quad 0]^{Tr},$$

$$\Delta_3 = [2\iota m_5 s_5 \quad \eta_1^* R_3 \xi \quad 0 \quad -1 \quad 0 \quad 0 \quad 0]^{Tr},$$

$$\Delta_4 = [2\iota m_5 c_5 \quad 0 \quad -2\iota\xi\eta_1^* m_5 \quad 0 \quad \iota m_5 \quad 0 \quad 0]^{Tr},$$

$$\Delta_i = [0 \quad d_i \quad h_i \quad -c \quad -cn_i \quad m_i k_i \quad m_i f_i]^{Tr}, i = 5,6,7,8$$

$$M_1 = \det(\Delta_1 \quad \Delta_3 \quad \Delta_2 \quad \Delta_5 \quad \Delta_6 \quad \Delta_7 \quad \Delta_8),$$

$$M_2 = \det(\Delta_4 \quad \Delta_3 \quad \Delta_2 \quad \Delta_5 \quad \Delta_6 \quad \Delta_7 \quad \Delta_8),$$

$$M_3 = \det(\Delta_4 \quad \Delta_1 \quad \Delta_2 \quad \Delta_5 \quad \Delta_6 \quad \Delta_7 \quad \Delta_8),$$

$$M_4 = \det(\Delta_4 \quad \Delta_1 \quad \Delta_3 \quad \Delta_5 \quad \Delta_6 \quad \Delta_7 \quad \Delta_8),$$

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